Part C: Volume/Surface Area of 3 Dimensional Shapes

- To calculate the volume/surface area of an object, all measures must in the same units. Therefore prior to performing any calculations convert all measurements to the same units.

- Composite figures can be created using 3 dimensional shapes.

- In the picture to the right the ice cream cone is made up of a cone on the bottom portion and a cylinder on the top.

- For prisms and cylinders, the volume can be found by multiplying the base area by the height.

- The surface area of a 3 dimensional object is the sum of the areas of all the outer faces of the object.

Example

A propane tank is in the shape of a cylinder with two hemispherical ends. The length of the cylindrical portion is 85 in. And the inside diameter of the tank is 25 in. Determine the surface area of the tank and the amount of propane, in gallons, that will fit inside the propane tank.
Solution

Part 1
Calculate the surface area of the cylinder portion excluding the top and bottom

Calculate the surface area of the semi sphere’s on each end. Since there are 2 pieces the surface area is equal to that of one complete sphere.

To find the total surface area we must add the two previous surfaces together.

Part 2
Calculate the volume of the cylindrical portion of the tank.

Calculate the volume of the hemispherical ends of the tank. The two hemispheres form a sphere with $r=12.5$ in.

To determine the total volume of the tank we must add the volume of the cylinder and sphere.

In order to provide our answer in gallons we must consult an online conversion tool. Therefore $49905.52$ in$^3$ converts to approximately 216 US gallons of propane.
Practice
1. Calculate the volume of each shape
   a) 
   ![Diagram of a sphere with a diameter of 18 cm]
   b) 
   ![Diagram of a triangular prism with dimensions 3.5 in., 2 in., and 0.5 in.]
   c) 
   ![Diagram of a right circular cylinder with dimensions 25 ft and 20 ft]
   d) 
   ![Diagram of a composite shape with dimensions 2.5 m, 4.2 m, 2.8 m, and 2.5 m]
2. Delio’s swimming pool is designed as shown

![Diagram of Delio's swimming pool]

a) Determine the amount of pool lining Delio will have to buy when the pool is installed.

b) Calculate the cost of the pool lining if it costs $3.89/ft²

c) Determine the maximum volume of water that the pool could hold.

d) Determine the volume of water that the pool could hold if it were filled to a point that is \( \frac{1}{4} \) m below the top of the pool.

3. Durval is adding 3 concrete steps to his back porch as shown. Each step is 80 cm long, 30 cm wide and 30 cm high.

![Diagram of Durval's concrete steps]

a) Determine the amount of cement Durval will require in order to build his steps.

b) Calculate the amount of surface area that Durval will have to smooth out on top of his steps (excluding the sides of the stairs)
## Formula Sheet – Student copy

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<thead>
<tr>
<th>Shape and Diagram</th>
<th>Volume / Perimeter</th>
<th>Surface Area / Area</th>
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<tbody>
<tr>
<td><strong>Cylinder</strong></td>
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<td><img src="image" alt="Cylinder Diagram" /></td>
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<td><strong>Square – based prism</strong></td>
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<td><img src="image" alt="Square Prism Diagram" /></td>
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<td><strong>Triangular Prism</strong></td>
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### Formula Sheet – Teacher copy

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| ![Cylinder Diagram](image) | \( V = (\text{area base})(H) \) \[\begin{align*} & = (\pi r^2)(H) \\
& = \pi r^2 H \end{align*}\] | \( \text{SA} = \text{top} + \text{bottom} + \text{side} \) \[\begin{align*} & = (\pi r^2) + (\pi r^2) + (2\pi r H) \\
& = 2\pi r^2 + 2\pi r H \end{align*}\] |
| **Square – based prism** | \( V = (\text{area base})(H) \) \[\begin{align*} & = (s)(s)(H) \\
& = s^2(H) \end{align*}\] | \( \text{SA} = \text{top} + \text{bottom} + \text{sides} \) \[\begin{align*} & = 2(s)(s) + 4(s)(H) \\
& = 2s^2 + 4s(H) \end{align*}\] |
| **Triangular Prism** | \( V = (\text{base area})(h) \) \[\begin{align*} & = \frac{1}{2} bh(h) \\
& = \frac{1}{2} blh \\
& = \frac{1}{2} \end{align*}\] | \( \text{SA} = \text{bases} + \text{sides} \) \[\begin{align*} & = 2\left(\frac{1}{2} bl\right) + ah + bh + cl \\
& = bl + ah + bh + cl \end{align*}\] |
| **Sphere**        | \( V = \frac{4}{3}\pi r^3 \) | \( \text{SA} = 4\pi r^2 \) |